

Stabilization of an arbitrary order transfer function with time delay H2 Controller

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ABSTRACT

Cyber physical system (CPS) has become the new generation of control industry because of the rapid advancement in computers and communications. Despite the advantages, the computations and manipulations have increased because of the complex network. This creates a lot of uncertainties such as input time delay and communication time delay which cause the performance degradation of a system. This paper proposed a robust H2 controller to overcome these effects in frequency domain. An enhanced way for linear time invariant stabilization of any order transfer function with time delay is adapted. Studied method is based on stability limit computation with H2 performance index. A numerical example is provided to illustrate the efficiency of the proposed method.

Keywords: Linear system, H2 Controller, Time delay, Frequency response, Robust control.



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1. Introduction:

Recently, time delay in a system has got a lot of attentions of the researchers from all over the world because of its significant impact on the performance of the systems. The significant sources of delay are computation and communication. The effects of delays become more swear in case of complex and large systems. Therefore, a suitable control technique is needed to overcome the influence of such delay on the system's performance. Applications of systems with delays [1,2] arise in physics, engineering [3], economics [4], biology, and operation research [5]. In data flow models, the reaction is delayed by drivers that gathered sensing, selection, response, perception, and programming delay might be supposed. Such delays are considered unaffordable for human nature and in traffic flow stability in the fast-growing world. Therefore, it is necessary to design a robust controller that may guarantee the stability and performance of a system under such circumstances.

Curiosity in considerations delays effects and projection of fixing controllers which account for delays is enhancing with the robustness of control systems. Generally, there is more effect of delays in distributed and interconnected systems [3,4], in which several controllers, sensors and actuators introduce several stochastic and deterministic delays. Delays may generate from the presence of mutual communication networks in connected systems, such as wireless networks and the internet [8,9,10]. We can also observe delays in the coordination of unmanned vehicle, tele-surgery, collaborative control multiple agents, tele-operation, adaptive combustion control, haptic and synchronization, chemical process with transport delays, sway control in cranes, and active vibration suppression [10,11,12].

H_2 optimal control theory is certainly in between precious tools for several variable control system design and analysis. Generally, H_2 optimal control problem sums to manufacturing a feedback controller for available plant to reduce H_2 norm of transfer function of control system that measures exogenous turbulences to output control variable. When such turbulences are given stochastic interoperation and considered to be freely zero mean white noise sequences of unit variance and that time H_2 optimal control problem is considered Linear Quadratic Gaussian problem, which marked initiate of control theory since 1960.

In standard configuration we considered the control system that contain controller and plant synthesis model. H_2 control problem common problem is studied for linear system discussed by rational transfer matrices, stable or necessarily is not mandatory. Basically, H_2 control problem contains internally stabilizing of control system while reduce H_2 norm of its transfer function. In the process control industry, H_2 controllers are considered the most crucial control utilized as an element. As a result, stability boundaries for H_2 control of systems with time delay have been broadly investigated. Previously, introduced the application of dimensional analysis in the tuning of H_2 controllers for first-order systems with a delay which was based on Buckingham's pi-theorem, while Olivera et.al presented the delay of the Hermite-Biehler theorem for quasi-polynomials which was the basis for deriving the stabilizing H_2 controller. The odd and even elements of denominator and numerator of the plant transfer function are required by the method used [13,14]. Moreover, Tan et.al investigated double integrator systems with time delay. All such methods depend on theorems that have been developed previously and the circumstance that plant parameters are already known [15, 16].

In light of above discussion, this paper solved the problem of stabilization for an arbitrary order transfer function in the presence of input time delay [18]. A unique solution to design H_2 controllers for delayed system is proposed by using frequency domain analysis [17, 18]. Here, the plant transfer function is decomposed into real and imaginary parts [21] and the stabilization of H_2 controllers is computed based completely upon decomposition. Compared to the results obtained in the previous works, no complex mathematical derivations are necessary; only the numerical frequency response of the plant transfer function is required. Besides, if the plant parameters are known, so same procedure can be used, and a complete analytical solution for all stabilizing H_2 gain values can be provided. The proposed controller can be designed according to the required gain and phase margin.

2. Internal Stability

The stability idea defines the system's ability to equilibrium return after the trivial disruption. Trivial disruption in the control system is not bearable at one position guide to boundless signals at another position (Position may refer to location). We think to ensure limited interior signals for all limited outer signals. To see only the stability of the closed-loop transfer function is insufficient. Though, the closed-loop transfer function is stable, internal signals may be limitless. To ensure limited interior signals, there must be stability in closed-loop system.

Definition 1:

Any two ends can be designated in a control system for interior stability test, but few selections are equivalent. One may select inputs $s(r)$ and $e'(r)$ and outputs $u(r)$ and $y(r)$ as shown in figure 1. Closed-loop system is stable if and only if transfer function matrix $G(s)$ elements forms $r(s)$ and $e'(s)$ to (s) and $y(s)$,

$$\begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = \mathbf{G}(s) \begin{bmatrix} r(s) \\ e'(s) \end{bmatrix} \quad 1$$

Where

$$\mathbf{G}(s) = \begin{bmatrix} \frac{H(s)D(s)}{1+H(s)D(s)} & \frac{H(s)}{1+H(s)D(s)} \\ \frac{D(s)}{1+H(s)D(s)} & \frac{-H(s)D(s)}{1+H(s)D(s)} \end{bmatrix} \quad 2$$

Review stability idea, there is stability in system if and only if all poles of closed loop transfer function are in open LHP. To compare with interior stability idea, stability concept is incomplete, because in feedback loop RHP zero pole cancellation is not in consideration. Here, zero pole cancellation describes that there is zero and pole at same point. Such could be in components like controller or in between two individual components such as in between controller and plant. There is few removable cancellations, like $r - 1/s - 1 = 1$. Before remove of cancellation, this transfer function is unstable. Few are not removeable cancellations, such as $d^{-s} - 1/s$. Commonly, such situation happened in time delay.

3. Performance Analysis:

In performance analysis, first see performance sensitivity,

$$\mathbf{R}(s) \cong \frac{1}{1+H(s)D(s)} \quad 3$$

Which study disturbance effect $e(s)$ on $u(s)$ output, or reference effect $r(s)$ on $d(s)$ error.

$$\mathbf{R}(s) = \frac{u(s)}{e(s)} = \frac{d(s)}{s(s)} \quad 4$$

Suppose, transfer function is represented by $T(s)$ from $r(s)$ to $u(s)$ reference. Closed-loop transfer function

$$\mathbf{T}(s) = \frac{u(s)}{r(s)} = \frac{H(s)D(s)}{1+H(s)D(s)} \quad 5$$

As

$$\mathbf{T}(s) = \mathbf{i} - \mathbf{R}(s), \quad 6$$

$T(s)$ is also known as complementary sensitivity function.

$R(s)$ Sensitivity function measures how sensitive $T(s)$ is disparity into $H(s)$ which is ratio of relative perturbation limit in $T(s)$, $\Delta T(s)/T(s)$ to perturbation relative in $H(s)$, $\Delta H(s)/H(s)$;

$$\lim_{\Delta H(s) \rightarrow 0} \frac{\Delta T(s)/T(s)}{\Delta H(s)/H(s)} = \frac{eT(s)H(s)}{eH(s)T(s)} = \mathbf{R(s)} \quad 7$$

Here, to see association among $R(s)$ and $T(s)$. Particular model is generally raised to as insignificant plant. Practically, one may take midpoint of all indeterminate plant as minor plant. Under such minor situation, it required to make $|R(xs)|$. Minor $|R(xs)|$ suggests the variations of reference and commotion have minor effect on output of system. These forms of system have advanced aptitude on commotion rejection. $|R(xs)|$ In algebra form for all frequencies may be equal to zero, that is perfect control. Although, practically it is incredible. In real system, $H(s)$ is generally strict.

$$\lim_{s \rightarrow \infty} \mathbf{R(s)} = \lim_{s \rightarrow \infty} \frac{1}{1+H(s)D(s)} = \mathbf{1} \quad 8$$

Commonly, $|T(xs)|$ should be as close as possible to accord. $A|T(xs)|$ near to unity represent large bandwidth of system and well tracking ability. Although, because of limitation $T(s) + R(s) = 1$, $|T(xs)|$ may be equal to unity only within limited frequency assortment.

Suppose performance difficulty, an important purpose of feedback is to maintain error in between $u(g)$ and $s(g)$ reference minor, in case of throughout system is pretentious by plant uncertainty and outer disturbance. Plant output $u(g)$. In way to performance enumerate minor index for error is studied. H_2 is mostly used performance index.

$$\min \int_0^{\infty} \mathbf{d}^2(\mathbf{g})dt = \min \|\mathbf{d}(\mathbf{g})\|_2^2 \quad 9$$

From above optimization solution, controller parameters and structure can be resolute. If $s(r)$ reference is known, then $Q(s) = r(s)$ weighting function is familiarized for reference normalization. So, $r'(s)$ system input is instinct. If instinct input, then output energy $d(g)$ is square of norm of system transfer function.

$$\|\mathbf{d}(\mathbf{g})\|_2 = \|\mathbf{Q(s)R(s)}\|_2, \frac{r(s)}{Q(s)} = \mathbf{1} \quad 10$$

Although, in Laplace domain performance index H_2 can be

$$\min \|\mathbf{Q(s)R(s)}\|_2 \quad 11$$

Suppose that $\min \|Q(s)R(s)\|_2 \rightarrow \delta$ containing δ constant effect into $Q(s)$, here we can represent index as

$$\|\mathbf{Q(s)R(s)}\|_2 < \mathbf{1}. \quad 12$$

Once more, $Q(s)$ is familiarised for system input normalization. If input is energy bounded signal by unity, the output energy is limited by the square of infinity norm of system transfer function.

$$\sup_{r(g)} \|d(g)\|_2 = \|Q(s)R(s)\|_\infty, \left\| \frac{r(s)}{Q(s)} \right\|_2 \leq 1$$

Here, we noticed that to indulge intended problem in incorporated mathematical from, weighting function is familiarised to performance index. We considered that $r(s)$ reference is produced by $r'(s)$ passing through $Q(s)$. $Q(s)$ is known as performance weighting function, because gained performance be contingent on prime of $Q(s)$. Norm selection is not crucial. Generally, performance index selection is not crucial. Basic purpose for control system project is to gain controller with required response. Adjustment characteristic in control system means that performance or norms directories may vary to high extent, gained replies are almost similar. The project needs may be gained to apply various norms and performance directories. Relatively, it is must to select performance index or norm which is easy in mathematics.

In several industries, engineers mostly distresses regulator problem. In such purpose, gained system generally has well capability for rejection of disturbance. At same extant, the purpose contained a condition on well tracking aptitude. Generally, servomechanism and regulator problem have common benefits, because disturbance effect $e(s)$ on $u(s)$ output is as that $r(s)$ reference on $d(s)$ tracking. We can discuss result by complementary sensitivity function and sensitivity function. For controller gain with commotion refusal, is required to reduce $|R(x_s)|$. There is constraint $R(s) = 1 - T(s)$ for throughout control systems. For $|R(x_s)|$ minimization is to build $|T(x_s)|$ near to 1. We can say this is controller with good capability.

Here, system type and asymptotic tracking property has been discussed, and relation of sensitivity function and asymptotic tracking property has been highlighted. Figure 1 is representing complimentary sensitivity function.

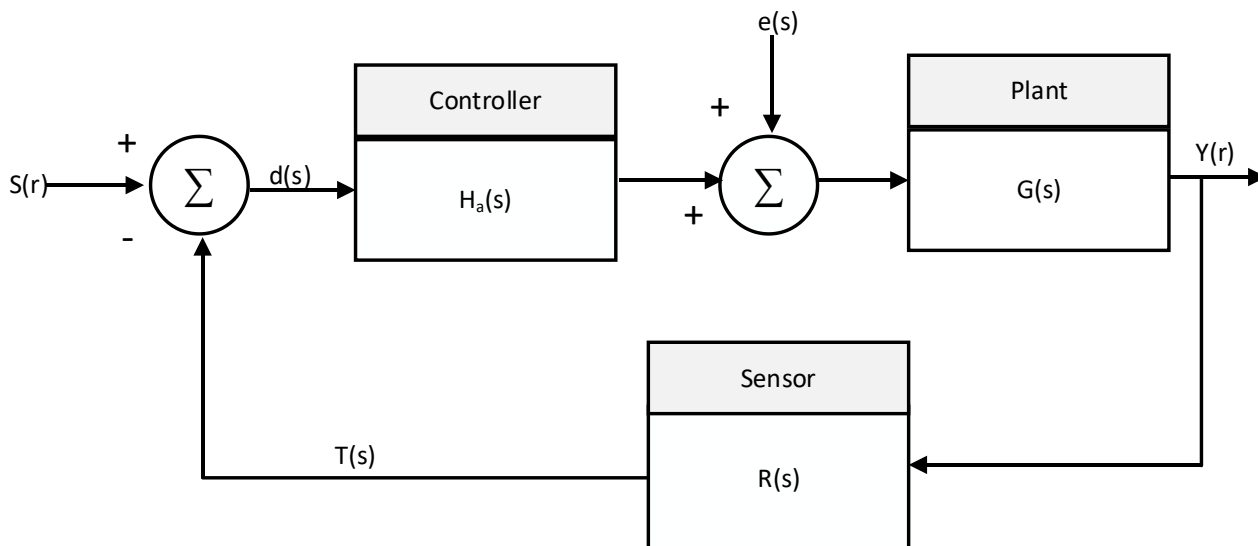


Figure 1: Feedback control system with complimentary function and sensitivity complimentary function

4. Stabilization and H₂ Controller Design

Suppose system with plant function as before, with H₂ controller is as

$$\mathbf{H}_a(\mathbf{s}) = \mathbf{X}_q + \mathbf{X}_e \mathbf{s} \quad 14$$

Similarly as in H2 controller, for finding stabilizing values of X_q and X_r in xs term, designed equation is

$$\Delta(\mathbf{x}\mathbf{s}) = \mathbf{1} + (\mathbf{X}_q + (\mathbf{X}_s \mathbf{s}) \mathbf{x})(\mathbf{S}_a(\mathbf{s}) + \mathbf{j}\mathbf{I}_a(\mathbf{s})). \quad 15$$

Now, for $\Delta(xs)$ expanding and adjusting zero product.

$$\mathbf{S}_\Delta(\mathbf{s}) + \mathbf{I}_\Delta(\mathbf{s}) = \mathbf{0} \quad 16$$

Where,

$$\mathbf{S}_\Delta(\mathbf{s}) = \mathbf{1} + \mathbf{X}_a \mathbf{S}_a(\mathbf{s}) - \mathbf{X}_e \mathbf{s} \mathbf{I}_a(\mathbf{s}) \quad 17$$

Now

$$\mathbf{I}_\Delta(\mathbf{s}) = \mathbf{X}_a \mathbf{I}_a(\mathbf{s}) - \mathbf{X}_e \mathbf{s} \mathbf{S}_a(\mathbf{s}) \quad 18$$

When imaginary and real parts are about equal to zero, then

$$\mathbf{X}_a(\mathbf{S}_a(\mathbf{s})) + \mathbf{X}_e(-\mathbf{s}\mathbf{I}_a(\mathbf{s})) = -\mathbf{1} \quad 19$$

$$\mathbf{X}_a(\mathbf{I}_a(\mathbf{s})) + \mathbf{X}_e(\mathbf{s}\mathbf{S}_a(\mathbf{s})) = \mathbf{0} \quad 20$$

However, there is requirement to solve following system

$$\begin{bmatrix} \mathbf{S}_a \mathbf{s} & -\mathbf{s}\mathbf{I}_a(\mathbf{s}) \\ \mathbf{I}_a \mathbf{s} & \mathbf{s}\mathbf{S}_a(\mathbf{s}) \end{bmatrix} \begin{bmatrix} \mathbf{X}_a \\ \mathbf{X}_e \end{bmatrix} = \begin{bmatrix} -\mathbf{1} \\ \mathbf{0} \end{bmatrix} \quad 21$$

From above equation, we gain

$$\mathbf{X}_a(\mathbf{s}) = -\frac{\mathbf{S}_a(\mathbf{s})}{|\mathbf{H}_a(\mathbf{x}\mathbf{s})|^2} \quad 22$$

Now

$$\mathbf{X}_e(\mathbf{s}) = \frac{\mathbf{I}_a(\mathbf{s})}{\mathbf{s}|\mathbf{H}_a(\mathbf{x}\mathbf{s})|^2} \quad 23$$

Note

$$\mathbf{X}_e(\mathbf{s}) = -\frac{1}{\mathbf{s}^2} \mathbf{X}_c(\mathbf{s}) \quad 24$$

At $s = 0$ equation (21) becomes

$$\begin{bmatrix} \mathbf{S}_a(0) & \mathbf{0} \\ \mathbf{I}_a(0) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{X}_a \\ \mathbf{X}_e \end{bmatrix} = \begin{bmatrix} -\mathbf{1} \\ \mathbf{0} \end{bmatrix} \quad 25$$

To solve above equation we gained X_e is freely,

$$\mathbf{X}_a = -\frac{1}{\mathbf{S}_a(0)} \quad 26$$

$$\mathbf{I}_a(\mathbf{0}) = \mathbf{0} \quad 27$$

In equation 26 lower bound for type zero plant is non-zero, then $S_a(0) = \infty$

$$\mathbf{X}_a = \mathbf{0} \quad 28$$

It is important to note that for throughout real plants $S_a(0)$ will hold in equation (27) and will be similar to plant DC gain.

Repeatedly, we require upper bound s_i , where $X_a(s) = X_a(0)$. System will mollify at critical frequency

$$\angle \mathbf{H}_a(\mathbf{xs}) \in \begin{cases} \left[-\pi, -\frac{3\pi}{2} \right), & \text{if } \mathbf{X}_a(\mathbf{0}) > \mathbf{0} \\ \left[-\frac{3\pi}{2} \right], & \text{if } \mathbf{X}_a(\mathbf{0}) = \mathbf{0} \\ \left(-\frac{3\pi}{2}, 2\pi \right], & \text{if } \mathbf{X}_a(\mathbf{0}) < \mathbf{0} \end{cases} \quad 29$$

$X_a(0) = 0$ Condition is mollify for higher system

Same case has been preceded as in H2 case. Here equation (14) has been used as controller function. For X_a and X_e values gain, equation (13) has been decayed in imaginary and real parts, and then auxiliary into equation (22) and (23).

5. Simulation Example

This section presents a numerical example to demonstrate the effectiveness of the proposed method.

Consider a paper making machine (PMM) [22] which is separated into five different parts: table and press, head, reel, calendar stocks and drying. There is stock system, while fibres have been dipped in water. In mixing tank interruption is carried. In head box and mixing tank, we mixed thick stock with the help of reused water. Diluted solution is delivered by head box to wire with tiny free holes. Such wires repeatedly transfers over table, and through wire maximum water is drained. On fibres there is wet mat produced by drained water. After dry and pressing process this wet mat forms a paper or becomes paper sheet.

In such system, there are several control subjects like, basis weight, steam pressure, moisture content and consistency and here basis weight is an important subject. In mechanics identification and analysis there developed low order model for control of basis weight.

$$\mathbf{H}(s) = \frac{5.15}{1.8s+1} - e^{2.8s} \quad 30$$

Here $X = 5.15$, $1.8s$ is constant coefficient of pole and $\theta = 2.8$ is time delay we gain following H₂ controller

$$\mathbf{W}(s) = \frac{(1.8s+1)(1.4s+1)}{5.15(\rho s+1)^2} \quad 31$$

And algebra gives such H₂ controller

$$D(s) = \frac{(1.8s+1)(1.4s+1)}{5.15[\rho^2 s^2 + (2\rho+1.4)s]} \quad 32$$

Smith predictor is

$$S(s) = \frac{(1.8s+1)(1.4s+1)}{5.15[\rho^2 r^2 + (2\rho+1.4)s]} \quad 33$$

Here, $\rho = 0.4\theta$.

At $g = 0$ a unit step is added and with -0.1 magnitudes a step load is added at $g = 50$. In this result system reply will be steady and fast. Here we take practically measured system response and theoretically find set of H2 controller as gain gap is equal or greater than 2 while phase gap is equal or greater than $\frac{\pi}{4}$. Virtual dynamic signal analyser is helpful in measuring frequency system response with 0.1 Sec. delay measurement. To verify the effectiveness of proposed controller we did simulation using MATLAB, in our control system we used $S(r)$ as input, $d(s)$ as error, $T(s)$ complimentary function and $e(s)$ as error and $y(r)$ as output. Results have been explained in Figure 2 and 3.

From motor input voltage frequency response has been measured. Bode, root locus and step response of the system have been shown in Figure 2(a), 2(b) and 2(c), respectively. Here 2(a) representing root locus for stability margin of bias weight under 0.1 delays. 2(b) is representing magnitude and phase plot of bias weight under 0.1 delay and 2(c) is representing step response of bias weight under 0.1 delay.

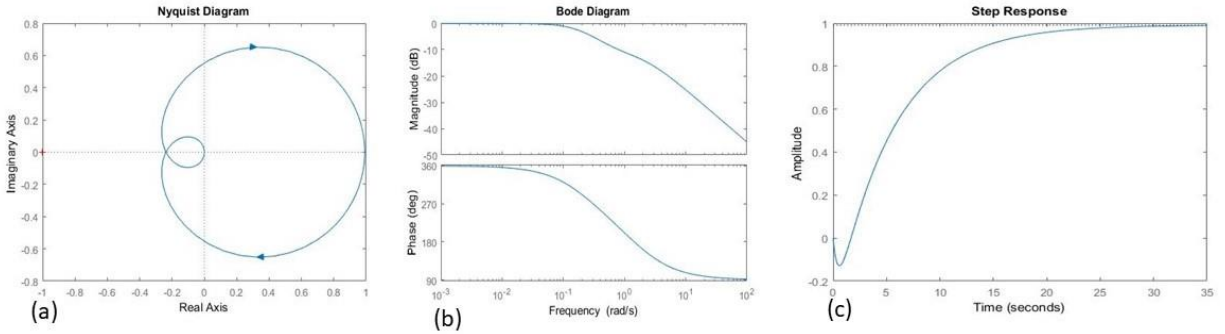


Fig. 2: Response of the System Response

First computed general stability boundary locus, like $\theta = 0$ and $HM = 1$. For satisfaction of gain margin, in controller designing first adjusted $\theta = 0$ and $l = 2$ in equation 18. From given frequency response, we may abstract imaginary and real value at S and supernumerary into equation (22) and (23) for stability border. For phase margin satisfaction there is repetition in procedure with $\theta = \frac{\pi}{4}$ and $E = 1$ as shown in Figure 3.

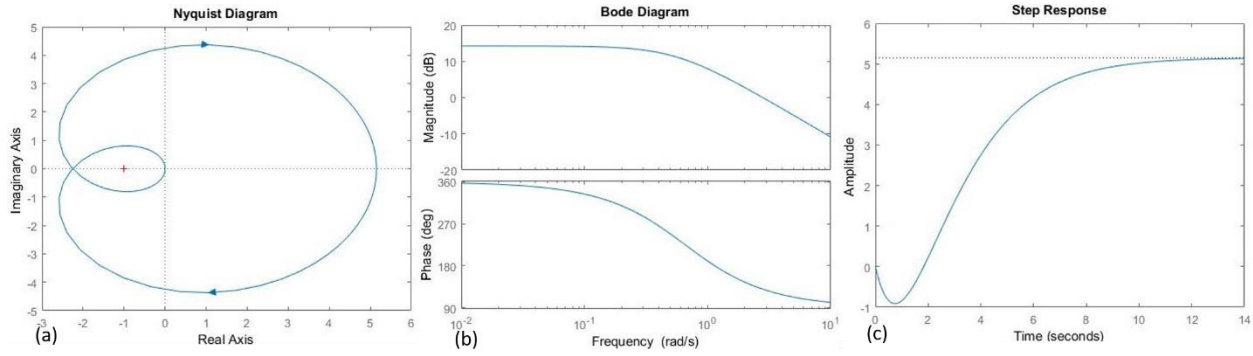


Fig. 3: Response of Feedback control system with proposed Controller.

Region gained from area intersection under results consists of stable value of X_q and X_s . For results verification, selected $X_s = 4.0$ and $X_q = 2$ and checked system response. So bode plot, root locus and step response of the feedback control system when H2 controller is applied, have been shown in Figures 3(a), 3(b) and 3(c), respectively. These figures show that the stability of the PMM has achieved after the implementation of this controller.

6. Conclusion

In this article, we presented an efficient and simple way of designing H₂ controller for the stabilization of a transfer function. We also studied a method for control system using time delay coordinates. The procedure is depending on frequency response of plant with no restriction on system order. The proposed strategy is verified in MATLAB simulation to emphasize the step response of system without controller and with the proposed controller simultaneously. These results show that the proposed method is easier, faster and non-complicated than previous existing methods. In this article we also evaluated algorithm for a system which require definite phase and gain margin.

Author's Contribution: Zahoor Ahmed & Muhammad Nasir, Conceived the idea; Zahoor Ahmed & Muhammad Muzamil Aslam., Designed the simulated work or acquisition of data; Muhammad Nasir & Muhammad Muzamil Aslam Executed simulated work, data analysis or analysis and interpretation of data and wrote the basic draft; Zahoor Ahmed, Did the language and grammatical edits or Critical revision

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